

The exam is open book open note.

SHOW NEAT, WELL ORGANIZED, ORDERED WORK.

1. Basic relativistic relations (5 pts):
Write out the two postulates of relativity and also the name of the experiment initially verifying that there is no "Ether".

1) Laws of Phys same for all
obs in inertial ref frames

2) speed of light same - all obs

& Michelson Morley

2. Given the matrix multiplying the column vector below, Determine the value for A.
(Show work explicitly—to determine A, this is one line) 5 pts.

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \\ 4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} n \\ A \\ m \end{pmatrix} \quad A = 3 \times 0 + 3 \times 2 + 1 \times 1 = 7$$

3. Use the relativistic statement for Kinetic Energy and show that for $v \ll c$, that the relativistic statement reduces to the classical formula. 15 pts

$$\begin{aligned} KE &= (\gamma - 1) m_0 c^2 \\ &= \left[\left(1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} - 1 \right] m_0 c^2 \\ &\approx \left[1 - \left(-\frac{1}{2} \right) \frac{v^2}{c^2} - 1 \right] m_0 c^2 \\ &\approx \frac{1}{2} m_0 v^2 \end{aligned}$$

4. In a relativistic science experiment, Doc Brown drives an ID Buzz past you at a speed of 39.3395 m/s. You are passed at $t=0.000000$. (20 pts)
- a) You observe 1.000000s time intervals on your watch. What is the difference between your interval reading and what you see on Doc Brown's watch. Keep whatever precision is needed here to display relativistic effects to at least three digits in the difference.



expand }
 $\gamma - 1$

$$T = \gamma T_0 \quad \Delta T = (\gamma - 1) T_0$$

$$\beta = \text{small} \\ T_0 = 1.000 \text{ s}$$

$$\gamma - 1 = \frac{1}{\sqrt{1 - \beta^2}} - 1$$

$$\approx \frac{1}{1 - \frac{1}{2}\beta^2} - 1$$

$$\approx \cancel{1} + \frac{1}{2}\beta^2 - \cancel{1}$$

$$\Delta T = \frac{1}{2} \beta^2 * 1 \text{ sec} = 8.60 \times 10^{-15} \text{ s}$$

- b) How fast should the ID Buzz be moving (windows open, with Einstein sticking his head out the window) so that you see the your clock interval read 1.0000s while YOU see Doc Brown's clock interval read 0.30000s. (you see Doc's clock running slow compared to yours)

$$T = \gamma T_0 \leftarrow 0.3 \quad \gamma = 3.33$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$= 0.954$$

$$V = 2.86 \times 10^8 \text{ m/s}$$

5. In problem 4b You observed ID Buzz moving with speed in part b, and now, your good friend and lab partner, Marvin the Martian, is going to fly by (YOU) in the same direction (+x) at a speed of 0.600000 c. (10 pts)



What velocity does Marvin the Martian observe for the Doc Brown in his ID Buzz?

ID
Buzz



$$V_{1x} = 0.954c$$

$$V = 0.600c$$



$$V_{2x} = \frac{V_{1x} - V}{1 - \frac{v}{c} V_{1x}}$$

$$= \frac{0.954c - 0.6c}{1 - 0.6 \times 0.954}$$

$$= 0.828c$$

$$= 2.48 \times 10^8 \text{ m/s}$$



6. Wile E Coyote has attached an Arduino constructed blinky light to the Road Runner in order to track his speed. At rest the blinky light emits a wavelength of 700nm. Mr. Coyote will shoot the road runner out of a cannon at $0.80000c$ in the positive x direction. (25)

- a) What is the frequency of the blinky light at rest?



$$\begin{aligned} \nu_0 &= \frac{c}{\lambda} \\ &= 4.29 \times 10^{14} \text{ Hz} \end{aligned}$$

- b) What **frequency** will Wile E Coyote observe as the fast moving Road Runner passes through the point at $x = +1.00\text{km}$ -----ahead and moving away. (y and z are zero).

$$\theta = 0$$

$$\gamma = 1.67$$



$$\begin{aligned} \nu &= \frac{\nu_0}{\gamma(1 + \frac{v}{c} \cos \theta)} \\ &= 1.43 \times 10^{14} \text{ Hz} \end{aligned}$$

- c) At $x = -1.00\text{km}$ (again y and z are zero)—what frequency is observed by Dr. Coyote?

$$\theta = 180^\circ$$

$$\nu = 1.28 \times 10^{15} \text{ Hz}$$

- d) Our foul/fowl friend has been moving in the direction of the + x axis, and still is. As he passes through $y = 1.00\text{km}$ (x and z are zero), what frequency is observed by Dr. Coyote?

$$\theta = 90^\circ$$

$$\nu = 2.57 \times 10^{14}$$

7. In the official placement on the periodic table, the Marvel Comic book/street named element "vibranium" is going to be officially named **T'Challium**---for obvious reasons (Wakanda forever). T'Challium has a rest mass of $286.4 \text{ GeV}/c^2$ (20 pts)

a) T'Challium is an excellent material to use in studies in an accelerator.
What speed must an atom of T'Challium have in order to achieve γ (gamma) = 2.000 ?

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.866$$

$$v = 2.6 \times 10^8 \text{ m/s}$$

b) What is the **total** energy the atom must have from part a to move that speed (use any units you like)?

$$\begin{aligned} E &= \gamma (286.4) \\ &= 572.8 \text{ GeV}/c^2 \end{aligned}$$

c) What is the kinetic energy of the atom (moving as it does in part a)?

$$\begin{aligned} KE &= (\gamma - 1) (286.4) \\ &= 286.4 \text{ GeV}/c^2 \end{aligned}$$